

Sivers Mechanism for Gluons

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We study T-odd gauge invariantly defined T-odd unintegrated gluon densities in QCD. The average \perp momentum of the gluons in a \perp polarized nucleon can be nonzero due to the Sivers mechanism. However, we find that the sum of the transverse momenta due to the Sivers mechanism from gluons and all quark flavors combined vanishes, i.e. $\langle \mathbf{k}_{\perp, \mathbf{g}} \rangle + \sum_q \langle \mathbf{k}_{\perp, \mathbf{q}} \rangle = 0$.

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I. INTRODUCTION

Recent experiments performed by the HERMES collaboration have demonstrated that a small but nonzero Sivers mechanism for transverse single-spin asymmetries exists [1]. Although the presence of such effects has been conjectured long ago [2], only more recent studies [3] have clarified the role of final and initial state interactions as a source of such T-odd asymmetries at the parton level and have thus demonstrated that a nonzero Sivers mechanism is possible in QCD. Following these insights, several model estimates for the Sivers asymmetry have been made [4] but only few theoretical constraints exist regarding the anticipated magnitude or even the sign of the resulting asymmetries in QCD. The main purpose of this letter is to derive a constraint on the Sivers mechanism for gluons.

For the final (initial) state interactions, previous results for the Sivers mechanism for quarks can be summarized in a relatively compact form by means parton densities with T -odd contributions. In the case of unintegrated quark densities one finds [5, 6, 7]

$$q(x, \mathbf{k}_{\perp}, \mathbf{s}_{\perp}) = \int \frac{dy^- d^2 \mathbf{y}_{\perp}}{16\pi^3} e^{-ixp^+ y^- + i\mathbf{k}_{\perp} \cdot \mathbf{y}_{\perp}} \langle p | \bar{q}(y^-, \mathbf{y}_{\perp}) \gamma^+ [y^-, \mathbf{y}_{\perp}; \infty^-, \mathbf{y}_{\perp}] [\infty^-, \mathbf{0}_{\perp}; 0^-, \mathbf{0}_{\perp}] q(0) | p \rangle. \quad (1)$$

We use light-front (LF) coordinates, which are defined as: $y^{\mu} = (y^+, y^-, \mathbf{y}_{\perp})$, with $y^{\pm} = (y^0 \pm y^3)/\sqrt{2}$. In all correlation functions, $y^+ = 0$ and we therefore do not explicitly show the y^+ dependence. The path ordered Wilson-line operator from the point y to infinity is defined as

$$[\infty^-, \mathbf{y}_{\perp}; y^-, \mathbf{y}_{\perp}] = P \exp \left(-ig \int_{y^-}^{\infty} dz^- A^+(y^+, z^-, \mathbf{y}_{\perp}) \right). \quad (2)$$

The specific choice of path in Ref. [6] reflects the FSI (ISI) of the active quark in an eikonal approximation. The complex phase in Eq. (1) is reversed under time-reversal and therefore T-odd PDFs may exist [6]. It should be emphasized that the presence of small x and UV divergences and the necessary renormalizations prevent us from literally interpreting these densities as number densities [8]. However, in this work we will only study the resulting average value of \mathbf{k}_{\perp} integrated over all x , where most of these divergences are expected to cancel.

Naively, the single spin asymmetry seems to be absent in light-cone gauge $A^+ = 0$, since the Wilson lines in Eq. (1) are in the x^- direction and therefore $\int dz^- A^+ = 0$. Without the phase factor any single spin asymmetry vanishes due to time reversal invariance. This apparent puzzle has been resolved in Ref. [5], where it has been emphasized that a truly gauge invariant definition for unintegrated parton densities requires closing the gauge link at $x^- = \infty$, i.e. a fully gauge invariant version of Eq. (1) reads (Fig. 1)

$$q(x, \mathbf{k}_{\perp}, \mathbf{s}_{\perp}) = \int \frac{dy^- d^2 \mathbf{y}_{\perp}}{16\pi^3} e^{-ixp^+ y^- + i\mathbf{k}_{\perp} \cdot \mathbf{y}_{\perp}} \langle p | \bar{q}(y^-, \mathbf{y}_{\perp}) \gamma^+ [y^-, \mathbf{y}_{\perp}; \infty^-, \mathbf{y}_{\perp}] [\infty^-, \mathbf{y}_{\perp}, \infty^-, \mathbf{0}_{\perp}] [\infty^-, \mathbf{0}_{\perp}; 0^-, \mathbf{0}_{\perp}] q(0) | p \rangle. \quad (3)$$

In all commonly used gauges, except the light-cone gauge, the gauge link at $x^- = \infty$ is not expected to contribute to the matrix element, since the gauge fields are expected to fall off rapidly enough at ∞ . However, this is not true in light-cone gauge and therefore it has been suggested in Ref. [5] that, in light-cone gauge, the entire single-spin asymmetry arises from the phase due to the gauge link at $x^- = \infty$

$$q(x, \mathbf{k}_{\perp}, \mathbf{s}_{\perp}) = \int \frac{dy^- d^2 \mathbf{y}_{\perp}}{16\pi^3} e^{-ixp^+ y^- + i\mathbf{k}_{\perp} \cdot \mathbf{y}_{\perp}} \langle p | \bar{q}(y^-, \mathbf{y}_{\perp}) [\infty^-, \mathbf{y}_{\perp}, \infty^-, \mathbf{0}_{\perp}] \gamma^+ q(0) | p \rangle \quad (A^+ = 0). \quad (4)$$

The main purpose of this letter is to generalize these results to T-odd parton distributions for the glue and to discuss the consequences for the net Sivers effect.

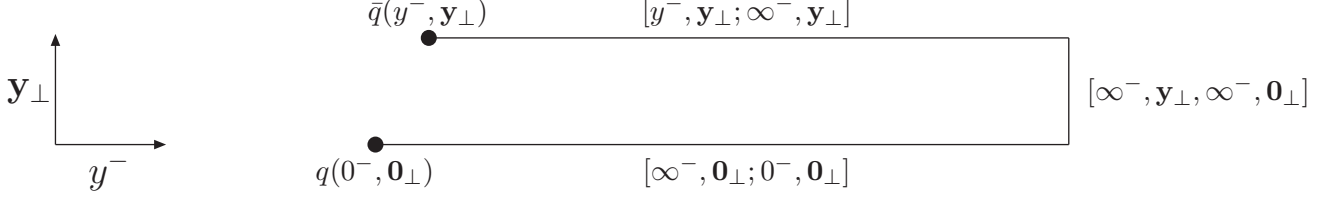


FIG. 1: Illustration of the gauge links in Eq. (3).

II. THE UNINTEGRATED GLUON DENSITY

The naive (not gauge invariant and not accounting for T-odd effects) definition for the unintegrated unpolarized (i.e. gluons unpolarized) gluon density reads (see also Refs. [9, 10], where a decomposition into Lorentz components is provided)

$$xg(x, \mathbf{k}_\perp, \mathbf{s}_\perp) \equiv \int \frac{dy^- d^2 \mathbf{y}_\perp}{8\pi^3 p^+} e^{-ixp^+ y^- + i\mathbf{k}_\perp \cdot \mathbf{y}_\perp} \langle p | \text{tr} (F^{+i}(y) F^{+i}(0)) | p \rangle,$$

where $F^{\mu\nu}$ is the gluon field strength tensor. Throughout this paper, we consider states $|p\rangle$, which are polarized in the \perp direction, but suppress the spin label in the state in order to shorten the notation.

In analogy with gauge invariant parton densities for the quarks, we now introduce gauge invariant gluon densities by augmenting Eq. (5) with appropriate (future-pointing) Wilson lines

$$xg(x, \mathbf{k}_\perp, \mathbf{s}_\perp) = \int \frac{dy^- d^2 \mathbf{y}_\perp}{8\pi^3 p^+} e^{-ixp^+ y^- + i\mathbf{k}_\perp \cdot \mathbf{y}_\perp} \langle p | \text{tr} \left(\hat{F}^{+i}(y) [\infty^-, \mathbf{y}_\perp; \infty^-, \mathbf{0}_\perp] \hat{F}^{+i}(0) [\infty^-, \mathbf{0}_\perp; \infty^-, \mathbf{y}_\perp] \right) | p \rangle \quad (5)$$

where

$$\hat{F}^{+i}(y) = [\infty^-, \mathbf{y}_\perp; y^-, \mathbf{y}_\perp] F^{+i}(y) [y^-, \mathbf{y}_\perp; \infty^-, \mathbf{y}_\perp]. \quad (6)$$

The particular choice of Wilson line (here future-pointing) depends on the process to which the gluon distributions are applied. Throughout this paper we focus on distributions defined with future-pointing lines. Past-pointing lines can be trivially obtained by a time-reversal operation and yield the same result, but with opposite signs for the transverse single-spin asymmetry.

Similarly to the quark case, the gauge link at $x^- = \pm\infty$ is expected to contribute only in the light-cone gauge, where the gauge links along the light-cone vanish and we can thus replace $\hat{F}^{+i} \rightarrow F^{+i}$ in Eq. (5). In the following we will work in light-cone gauge, where $F^{+i} = \partial_- A^i$ and therefore

$$g(x, \mathbf{k}_\perp, \mathbf{s}_\perp) = i \int \frac{dy^- d^2 \mathbf{y}_\perp}{8\pi^3} e^{-ixp^+ y^- + i\mathbf{k}_\perp \cdot \mathbf{y}_\perp} \langle p | \text{tr} (A_i(y) [\infty^-, \mathbf{y}_\perp; \infty^-, \mathbf{0}_\perp] \partial_- A_i(0) [\infty^-, \mathbf{0}_\perp; \infty^-, \mathbf{y}_\perp]) | p \rangle \quad (7)$$

III. AVERAGE TRANSVERSE MOMENTUM

In order to evaluate the average transverse momentum we multiply Eq. (7) by \mathbf{k}_\perp and integrate over $d^2 \mathbf{k}_\perp$. The factor \mathbf{k}_\perp is replaced by a derivative w.r.t. \mathbf{y}_\perp . The term where this \perp derivative acts on $A_i(y)$ does not contribute in the end because of time reversal invariance. The only relevant contribution arises when the derivative acts on the gauge link at $y^- = \infty$, yielding

$$\bar{\mathbf{k}}_{\perp g}(x) \equiv \int d^2 \mathbf{k}_\perp \mathbf{k}_\perp g(x, \mathbf{k}_\perp, \mathbf{s}_\perp) = gf^{abc} \int \frac{dy^-}{4\pi} e^{-ixp^+ y^-} \langle p | A_{i,b}(y^-) \partial_- A_{i,c}(0) A_{\perp,a}(\infty^-, \mathbf{0}_\perp) | p \rangle. \quad (8)$$

Time reversal transformations reverse the orientation of the gauge string and we can therefore replace $A_{\perp,a}(\infty^-, \mathbf{0}_\perp) \rightarrow \alpha_{\perp,a}(\mathbf{0}_\perp)$, where

$$\alpha_{\perp,a}(\mathbf{0}_\perp) \equiv \frac{1}{2} (\mathbf{A}_{\perp\mathbf{a}}(\infty^-, \mathbf{0}_\perp) - \mathbf{A}_{\perp\mathbf{a}}(-\infty^-, \mathbf{0}_\perp)). \quad (9)$$

In particular we find for the net transverse momentum carried by the gluons due to the Siverts effect

$$\langle \mathbf{k}_{\perp g} \rangle \equiv \int dx \int d^2 \mathbf{k}_\perp g(x, \mathbf{k}_\perp, \mathbf{s}_\perp) = \frac{g}{2p^+} \langle p | f^{abc} (A_{i,b}(y^-), \partial_- A_{i,c}(0)) \alpha_{\perp,a} | p \rangle. \quad (10)$$

Eq. (10) is invariant under residual gauge transformations, which keep $A^+ = 0$. One of the applications of this result lies in the fact that Eq. (8) can be more directly evaluated in models (or calculations) of the light-cone wave function of hadrons than the starting equation (5). However, the main significance of this result will become more evident below when we combine Eq. (8) with the analog relation for quarks.

In Ref. [11] it was found that, in light-cone gauge, the average transverse momentum (integrated over x) of the active quark resulting from this definition can be related to the correlation between the colored quark density and the transverse gauge field at $x^- = \pm\infty$

$$\langle \mathbf{k}_{\perp q} \rangle = \int dx \int d^2 \mathbf{k}_\perp q(x, \mathbf{k}_\perp, \mathbf{s}_\perp) \mathbf{k}_\perp = -\frac{g}{2p^+} \left\langle p \left| \bar{q}(0) \gamma^+ \frac{\lambda_a}{2} q(0) \alpha_{\perp,a}(\mathbf{0}_\perp) \right| p \right\rangle. \quad (11)$$

From the condition that the light-cone energy is finite at $x^- = \pm\infty$, one can derive several constraints on the gauge field at $x^- = \pm\infty$ [11]. First it must be pure gauge $F_{12}^a |p\rangle = 0$, i.e. with appropriate boundary conditions (e.g. anti-symmetric) this implies

$$(\partial_1 \alpha_{2,a} - \partial_2 \alpha_{1,a} + g f^{abc} \alpha_{1,b} \alpha_{2,c}) |p\rangle = 0. \quad (12)$$

Secondly, the condition that $F_{+-}(\pm\infty^-, \mathbf{x}_\perp) |p\rangle = 0$ yields [11, 12]

$$\partial_i \alpha_{i,a}(\mathbf{x}_\perp) |p\rangle = -\rho_a(\mathbf{x}_\perp) |p\rangle \quad (13)$$

where $\rho_a(\mathbf{x}_\perp) = \int dx^- J_a(x^-, \mathbf{x}_\perp)$ and

$$J_a(x^-, \mathbf{x}_\perp) = -g f_{abc} A_b^i \partial_- A_c^i + g \sum_q \bar{q} \gamma^+ \frac{\lambda_a}{2} q. \quad (14)$$

If we now add up the total Siverts effect for quarks and gluons, we thus find

$$\langle \mathbf{k}_{1,g} \rangle + \sum_q \langle \mathbf{k}_{1,q} \rangle = -\frac{g}{2p^+} \langle p | J_a(0) \alpha_{\perp,a}(\mathbf{0}_\perp) | p \rangle, \quad (15)$$

This result is very interesting because it contains the same color charge density J_a (14) that also appears in the constraint equation for $\alpha_{\perp,a}$ (13). In the following we will demonstrate that the total transverse momentum (15) vanishes identically. For this purpose we first use translation invariance in the y^- direction to replace $J_a(0)$ by $\frac{1}{L} \rho_a(\mathbf{0}_\perp) = \frac{1}{L} \partial_i \alpha_{i,a}(\mathbf{0}_\perp)$ in Eq. (15), where L is the length in x^- of a fictitious ‘box’ that we introduce here for regularization purposes. For a target that is polarized in the \hat{y} direction we need to consider only the momentum in the \hat{x} direction since the momentum in the \hat{y} direction vanishes already for each piece separately. Combining the above results for the \hat{x} component we thus find

$$\langle \mathbf{k}_{1,g} \rangle + \sum_q \langle \mathbf{k}_{1,q} \rangle = -\frac{g}{2p^+ L} \langle p | (\partial_i \alpha_{i,a}) \alpha_{1,a} | p \rangle = -\frac{g}{2p^+ L} \langle p | (\partial_2 \alpha_{2,a}) \alpha_{1,a} | p \rangle = \frac{g}{2p^+ L} \langle p | \alpha_{2,a} \partial_2 \alpha_{1,a} | p \rangle \quad (16)$$

where we first used that the forward matrix element of $(\partial_1 \alpha_{1,a}) \alpha_{1,a} = \frac{1}{2} \partial_1 (\alpha_{1,a} \alpha_{1,a})$ vanishes and then, for the same reason, we replaced $(\partial_2 \alpha_{2,a}) \alpha_{1,a}$ by $-\alpha_{2,a} \partial_2 \alpha_{1,a}$. We now use the fact that α_\perp is pure gauge, i.e. acting on finite energy states we must have

$$\partial_2 \alpha_{1,a} |p\rangle = [\partial_1 \alpha_{2,a} - g f^{abc} \alpha_{1,b} \alpha_{2,c}] |p\rangle. \quad (17)$$

Upon inserting this result in Eq. (16) and utilizing again the fact that the forward matrix element of a total derivative $\alpha_{2,a} \partial_1 \alpha_{2,a} = \frac{1}{2} \partial_1 (\alpha_{2,a} \alpha_{2,a})$ vanishes we finally find

$$\langle \mathbf{k}_{1,g} \rangle + \sum_q \langle \mathbf{k}_{1,q} \rangle = -\frac{g^2}{p^+ L} \langle p | \alpha_{1,a} f^{abc} \alpha_{1,b} \alpha_{2,c} | p \rangle = 0 \quad (18)$$

due to the antisymmetry of the $SU(N)$ structure constant. While the presence of the final state interactions allows a nonzero Sivvers mechanism for the gluons as well as for each quark flavor, the net transverse momentum of all partons (quarks plus gluons) resulting from the Sivvers effect must vanish. In Ref. [11] a similar result was derived for QED. In the QED case the physics of this result is much more transparent because there one can solve the “finiteness conditions” explicitly and we refer the reader to that paper for more details.

The Sivvers effect arises because a parton can be transversely deflected when it traverses the Lorentz contracted gauge field from the spectators. The physical interpretation of our above result is that the transverse forces which the partons can exert on each other due to the gauge field interactions cancel each other when we sum over all partons. Such a result is very familiar from Newton’s 3^{rd} law where action=reaction implies that the net force on a multi-particle system with only internal forces vanishes. What we observe here is a similar effect, but we will not try to exploit this classical analogy here any further.

IV. SUMMARY

We have introduced gauge invariant unintegrated gluon densities with future-pointing gauge strings and derived relations between the resulting gluon-Sivvers effect and a correlator between the gluons density and the transverse gauge field at $x^- = \pm\infty$. While we considered here only unintegrated gluon densities with future-pointing gauge strings, the results can be easily generalized to distributions with past-pointing gauge strings by a time-reversal transformation. In particular the spin asymmetry simply changes sign for past-pointing strings. Whether future- or past-pointing strings should be used depends is not arbitrary but depends on the experimental conditions in which they are used. The results from this paper, with appropriate signs, should be applicable to either case.

While the net transverse momentum of the gluons arising from the Sivvers effect can be nonzero, we demonstrated that the net transverse Sivvers momentum from all quark flavors plus the gluonic piece combined vanishes. Such a result may be intuitively not so surprising, but it is nevertheless nontrivial when one starts from gauge invariantly defined unintegrated parton densities, where transverse momentum conservation is not evident. In fact, first ansätze to parameterize the gluon-Sivvers distribution [9] did not utilize such a constraint.

The main application of our result is that it provides an additional constraint on parameterizations of Sivvers distributions allows to relate the average Sivvers effect for different experiments (which may probe different flavors or the glue) to one another.

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